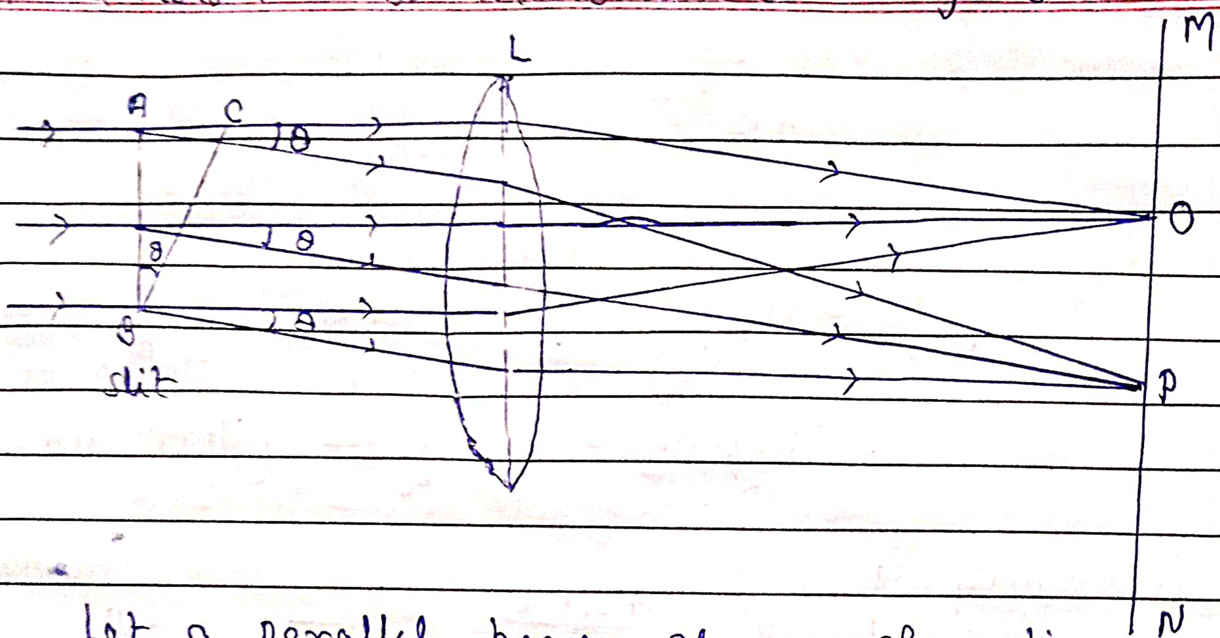
* Fraunhofer diffraction due to a single slit :-



Let a parallel beam of monochromatic light of wavelength λ be incident normally on a narrow slit AB of width l. The length of slit is along normal to the plane in figure. Every point of the slit will be source of secondary waves. which are all in the same phase on the slit. The parts of the secondary waves which procede normal to the slit will be converged at O by lens L to form what is known as pricipal maximum. The parts of the secondary waves which will meet at point P having an angle θ with normal, produce either min^m or max^m depending on their phase relationship. The path difference of rays from extreme points A and B of slit is $AC = l \sin\theta$.

Hence their phase difference $\delta = \frac{2\pi}{\lambda} \cdot l\sin\theta$

Let slit AB be divided into a large no. n, each of equal width. The amplitude of vibration at P due to each small width has same value say a. Hence phase difference between rays from two consecutive small width will be

$$\frac{1}{n}\left(\frac{2\pi}{\lambda} l\sin\theta\right) = d \quad (\text{let})$$

Hence resultant amplitude at P,

$$R = \frac{a \sin \frac{n\alpha}{2}}{\sin \frac{\alpha}{2}} = \frac{a \sin\left(\frac{\pi l \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi l \sin\theta}{n\lambda}\right)}$$

If $\frac{\pi l \sin\theta}{\lambda} = \alpha$ (let)

Then, $R = \frac{a \sin\alpha}{\sin\left(\frac{\alpha}{n}\right)} = \frac{a \sin\alpha}{\alpha/n}$ $\left[\because \frac{\alpha}{n} \text{ is very small}\right]$

$= \frac{na \sin\alpha}{\alpha} = \frac{A \sin\alpha}{\alpha}$ [Here, $na = A$] —— (i)

$\therefore$ Intensity at P,

$$I = R^2 = \frac{A^2 \sin^2\alpha}{\alpha^2} \quad \text{—— (ii)}$$

Principal Maxima :—

Eqn. (i) can be written as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right]$$

R will be max.^{m} when terms containing α become zero i.e.

$\alpha = \frac{\pi l \sin\theta}{\lambda} = 0 \quad \because \theta = 0°$

$\therefore$ $I_{max.} = R^2 = A^2$. It is the intensity of pricipal maxima which forms at O on the screen MN.

Position of minimum Intensity :—

From eqn. (i) or (ii) for minimum intensity,

$\sin\alpha = 0 \quad \therefore \alpha = \pm 1\pi, \pm 2\pi, \pm 3\pi, \cdots$

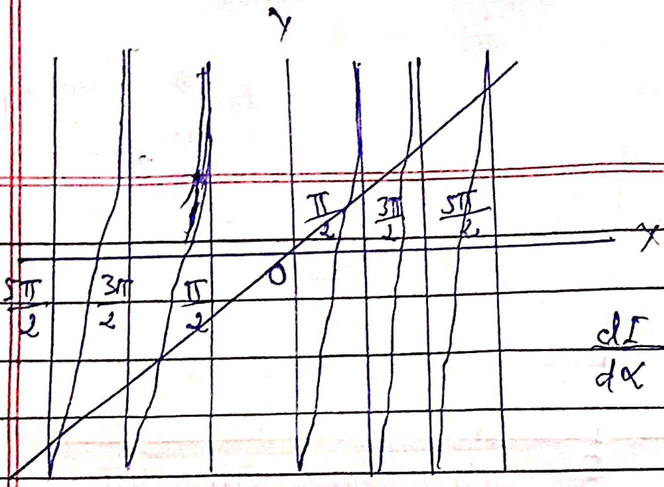
$= \pm m\pi$, $m = 1, 2, 3, \cdots$

$\therefore \frac{\pi l \sin\theta}{\lambda} = \pm m\pi$

or, $l \sin\theta = \pm m\lambda$ —— (iii)

Secondary maxima :—

Between two minima there is a secondary maximum. Their position can be found out using calculus method.



From eqn (ii),

$$I = I_{max} \frac{\sin^2\alpha}{\alpha^2}.$$

Diff. w.r. α

$$\frac{dI}{d\alpha} = 2 I_{max} \frac{\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

for maxima

Hence either, $\sin\alpha = 0$

or, $\alpha\cos\alpha - \sin\alpha = 0$

But $\sin\alpha = 0$ is the condition for minima.

Hence condition for secondary maxima is

$\alpha\cos\alpha - \sin\alpha = 0$

$\therefore \alpha = \tan\alpha$. ——— (iv)

The value of α satisfying eqn (iv) is obtained by graphical method when $y = \alpha$ and $y = \tan\alpha$. Curves are plotted on the same graph.

$y = \alpha$ curve is a straight line AB passing through origine and inclined at 45° with x-axis, $y = \tan\alpha$ eqn gives tangential curves having asymptotes at $\alpha = \pm\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ---$ as shown in fig. The points of intersection of these two curves gives those values of α for which we obtained secondary maxima.

If we consider intensity for principal maximum $I_{max} = 1$, then for first secondary max^m:

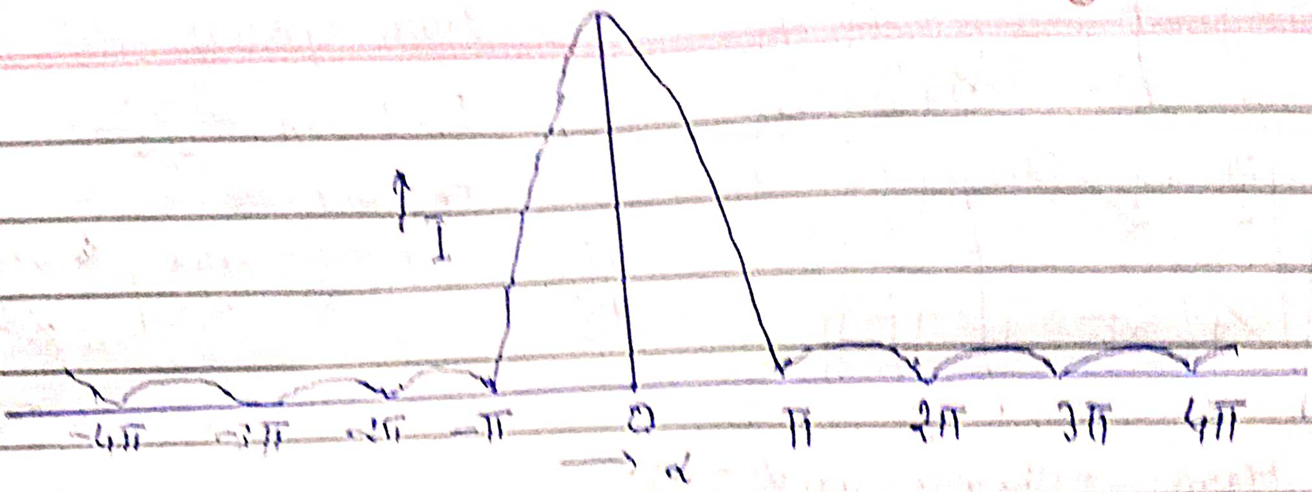
$\alpha = 3\pi/2$

Hence eqn. (ii)

$$I_1 = I_{max.}\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = 1 \times \frac{4}{9\pi^2} = \frac{1}{22.2}$$

Hence Intensity of first secondary max^m is $\frac{1}{22.2}$ times the intensity of principal max^m.

Intensity distribution curve for single slit is shown in below figure.



Principal maximum is formed at $\alpha = 0$ and minima are formed at $\alpha = \pm\pi, \pm 2\pi,$ —— between two minima there is a secondary maxima whose positions are

$$\alpha = \pm 1.430\pi, \pm 2.459\pi, \text{——}$$